COMPRESSION AND SHEAR WAVE SONIC VELOCITY MEASUREMENTS IN HARD ROCK

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Compression wave sonic velocity (V_p) is routinely measured in rock testing laboratories. Shear wave sonic velocity (V_s) measurement for further application to geomechanical studies is not routinely conducted. This paper outlines the establishment of a laboratory testing technique including waveform analysis for the determination of shear wave velocity.

The paper outlines the measurement of compressional and shear wave sonic velocities using ultrasonic pulse transmission technique, for several hard rock lithologies recovered during routine (NQ/HQ/PQ) exploration core drilling. Shear wave sonic velocities were measured using a pair of shear piezoelectric transducer elements. Measured shear wave sonic velocities are compared with fundamental and empirical formulas used to predict shear wave sonic velocity, in order to verify the method.

This paper discusses the need for an Australian Standard that includes a provision for the measurement of shear wave sonic velocity. Measured results are used to calculate dynamic moduli of rock samples and are compared with static moduli. The application of dynamic moduli to geotechnical characterisation of the rock mass is explored.

INTRODUCTION

A sonic, also referred to as an acoustic wave is a mechanical wave in which energy propagates through a medium via adiabatic compression and decompression. Propagation of acoustic waves through rock are measured in the field and in the laboratory setting, using different methods. The fundamentals of acoustic wave propagation through rock are consistent despite the change in environment and method of measurement. Acoustic velocity measurement is a record of the time required for an acoustic wave to travel a known distance, through a rock (Bassiouni, 1994).

Acoustic wave velocity can be used to evaluate porosity, lithology, dynamic moduli, clastic rock strength, rock anisotropy, underground cavities, and rock discontinuities. This non-destructive measurement has extensive application in geotechnical engineering. The frontier is continually being expanded by academics and professionals alike with papers detailing theoretical studies, field and laboratory results and their application in evaluating specific geotechnical characteristics.

Methods for measuring acoustic wave velocity in the field are extensive. Boreholes drilled during resource exploration provide a significant opportunity to capture acoustic wave velocity in the rock mass. In Australia – in the mining industry particularly – downhole geophysical surveys are routinely conducted to this end. A sonic tool is used to measure the compression and shear wave sonic velocity, from which dynamic moduli and formation evaluation is derived.

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Methods for measuring acoustic wave velocity in the laboratory are less extensive and are predominantly based on ultrasonic pulse transmission technique. The method uses multiple electrical components. Simply, a (transmitter) transducer generates a small amplitude acoustic wave which propagates through a rock sample of known size. The acoustic wave is received by a (receiver) transducer, and the time taken for propagation between the two transducers is captured by an oscilloscope. Acoustic wave velocity is calculated from the time taken to travel through the known distance. There is currently no Australian Standard for the measure of acoustic wave velocity in rock.

This paper presents a dataset of hard rock samples from the porphyry copper-gold Alpala deposit in North Ecuador. These samples were subject to acoustic wave velocity measurements in an Australian geotechnical laboratory – Strata Testing Services (STS). As part of a larger geotechnical testing program, ninety (90) samples were subject to compression wave sonic velocity measurements. Fifty six (56) samples were subject to shear wave sonic velocity measurements, with all samples tested in an unconfined condition. The method used to conduct these laboratory measurements and the results obtained are presented in this paper. The results are verified using fundamental and empirical equations. Dynamic moduli are calculated from the acoustic properties of the rock samples and are compared with the static moduli measured in the geotechnical laboratory.

LITERATURE REVIEW

Acoustic velocity is dependent on the elastic properties of rock (Bassiouni, 1994). Therefore, acoustic velocity can be used to measure elastic properties of rock. A basic introduction to elasticity and elastic wave propagation is presented next.

Stress and strain are basic engineering concepts. Rock elasticity can be quantified by measuring the change in shape and size of a rock (strain) that is subject to external forces of known magnitude over a known area (stress). In the context of this paper, stress is applied by the mechanical pulse of a piezoelectric element housed in a (compression or shear) transducer.

The acoustic waveform consists of multiple acoustic waves namely, compression, shear, Rayleigh and Stoneley waves. Compression waves have the highest velocity and Stoneley waves the slowest, due to the nature of wave propagation. Compression and shear waves are presented herein.

A mechanical pulse applied to an elastic body (rock) will cause it to instantaneously compress. The domain where particles are most compressed will displace away from the point of impact. The compression wave is transmitted through the elastic body via a series of compressions and dilations that occur along the direction of propagation.

Particle motion in a shear wave is perpendicular to the direction of propagation. A shear stress pulse acting on an elastic body yields a shear wave. In the context of this paper, shear stress is applied by the lateral mechanical oscillation of a piezoelectric element housed in a shear transducer. Figure 1 presents the ideal compression (a) and shear (b) wave propagation through a homogenous, elastic body.

In his book “Theory, measurement and interpretation of well logs”, Bassiouni derives the relationship between the shear and compressional wave velocities and the elastic constants: Young’s modulus (E), Poisson’s ratio (ν), bulk modulus (K) and shear modulus (G) (Bassiouni, 1994, p45-47). These relationships can be used to calculate the elastic constants from measurements of density (ρ), V_p and V_s. The fundamental equations are presented below and have been used to validate the measured shear wave sonic velocity and calculate dynamic moduli in this paper.

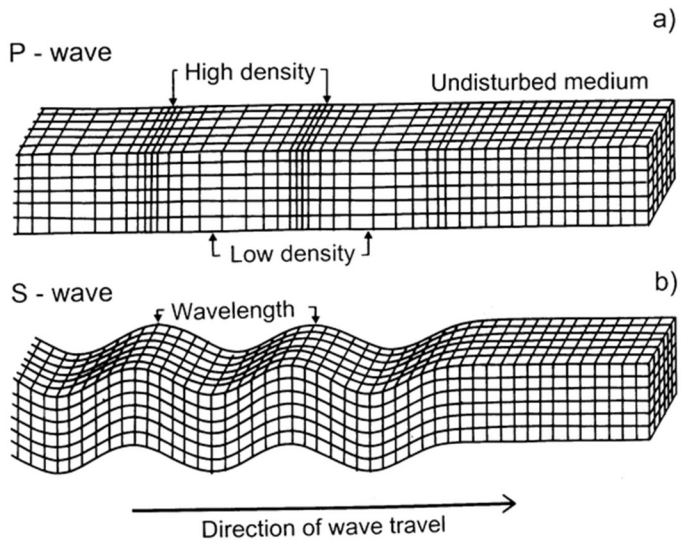

Figure 1: Compression (a) and shear (b) wave propagation through a homogenous, elastic body (Landstreet, 2009)

Elastic properties derived from equations of motion by Bassiouni 1994:

$V_p = [(K + 4/3)G/\rho]^{(1/2)}$ [Equation 3.18 (Bassiouni, 1994)]

$V_s = (G/\rho)^{(1/2)}$ [Equation 3.19 (Bassiouni, 1994)]

$V_p/V_s = [(2(1 - \nu) / (1 - 2\nu)]^2$ [Equation 3.23 (Bassiouni, 1994)]

Dynamic moduli equations

$G = V_s^2 \rho$ (Rearranging equation 3.19) (1)

$\nu = \frac{(2 - \frac{V_p^2}{V_s^2})}{-2(\frac{V_p^2}{V_s^2} + 1)}$ (Rearranging equation 3.23) (2)

$E = 2G(1 + \nu)$ (Rearranging equation 3.7) (3)

$K = G\,(\frac{V_p^2}{V_s^2} - \frac{4}{3})$ (Derived from equation 3.18) (4)

Christensen's equation is an empirical derived equation commonly used in hard rock environments to calculate shear wave sonic velocity (Firth and Elkington, 1999). The equation – presented below – is based on compression wave sonic velocity and density. This empirical equation will be used in conjunction with the equations of motion derived V_s calculation in order to validate the shear wave sonic velocity measurements presented in this paper.

$V_s = V_p\left[1 - 1.15((1/\rho + 1/\rho^3)/(e^{1/p}))\right]^{(3/2)}$ (Firth and Elkington, 1999) (5)

ρ – density (kg/m^3)

METHODOLOGY

A fundamental explanation of the ultrasonic pulse transmission technique used to measure the sonic velocities of rock is presented in this section. Figure 2 presents a schematic of the testing apparatus used to measure the sonic velocities of rock.

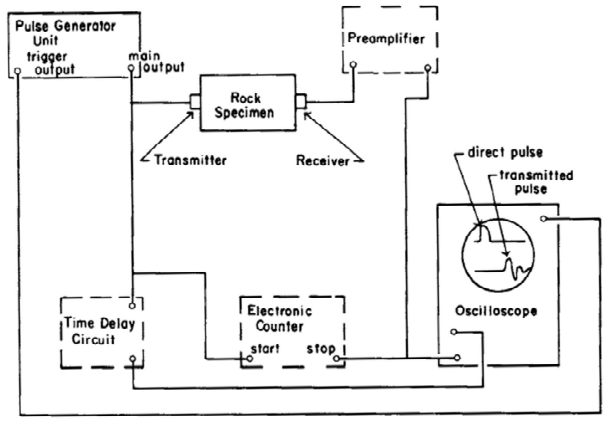

Figure 2: Schematic diagram of laboratory apparatus used to measure sonic velocity in rock (ASTM D2845-08)

Figure 3: HQ rock sample subject to sonic velocity measurements using (compression) transducers

An electrical pulse is generated through the pulse velocity generator at a particular frequency (stipulated in the relevant standards). This provides a source of electrical energy to the (transmitter) transducer, located at one end of the rock sample, and to the oscilloscope. This initial pulse is received by the oscilloscope and used as the trigger for the timing of the acoustic wave propagation.

The transmitter transducer converts the electrical energy supplied into mechanical energy, transmitting a consistent mechanical pulse into the rock specimen. The mechanical pulse from the transducer produces a rapid displacement at the end of the rock sample, creating an acoustic wave. This acoustic wave propagates through the rock sample.

The receiver transducer is connected to the opposite end of the rock specimen. Once the acoustic wave propagates through to the opposite end of the rock specimen the mechanical energy of the wave excites the piezoelectric element of the receiver transducer. The receiver transducer converts the mechanical energy back into electrical energy. This electrical pulse is displayed on the oscilloscope as the transmitted pulse or the 'arrival' of the sonic wave. Figure 4 presents an example of the oscilloscope display showing trigger and 'arrival' of the sonic wave.

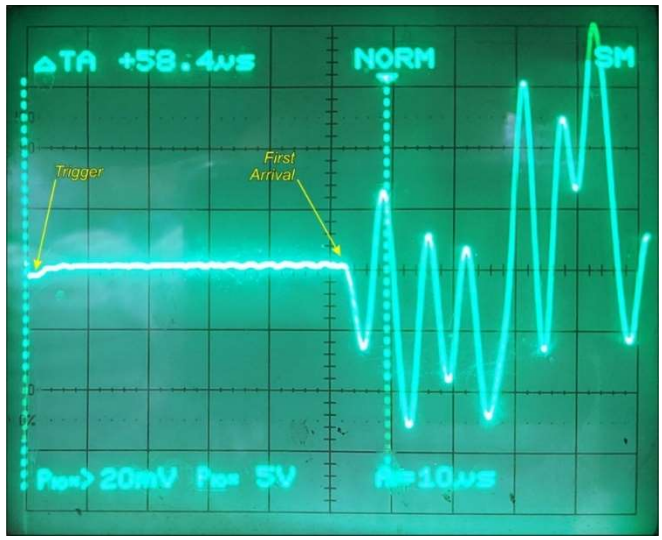

Figure 4: Oscilloscope display of transit time between trigger and sonic wave arrival

The laboratory technician, using the oscilloscope, can then easily determine the transit time, which is the time between the trigger (direct pulse) of the pulse velocity generator and the transmitted pulse (arrival of the sonic wave). The length of the rock specimen is measured at the start of the test. Allowing for the sonic velocity to be calculated simply by Equation 6.

$$V_p\ (m/s) = \frac{lengt\ \ of\ sample\ (mm)}{transit\ time\ (\mu s)} * 10^3 \tag{6}$$

RMS T224 Test Method

The RMS T224 test method framework was used for measuring the sonic velocities presented in this paper. Sections 3 to 6 of RMS T224 describe the method for measuring the compression wave sonic velocity (V_p) in rock. However, no provision for the measurement of shear wave sonic velocity (V_s) was made by the standard.

Measuring the shear wave sonic velocity

Two benchmark standards exist for measuring the sonic velocity of rock, these include:

1. The International Society for Rock Mechanics (ISRM) – Upgraded ISRM Suggested Method for Determining Sound Velocity by Ultrasonic Pulse Transmission Technique
2. American Society of Testing Methods (ASTM) – ASTM D2845-08

Table 1 presents a summary of test method attributes – for key test methods – that are relevant to this paper. In the authors' opinion the ASTM D2845-08 method is the most comprehensive and universal test method for determining the sonic velocity of rocks using ultrasonic pulse transmission technique.

Table 1: Summary of significant test methods

Test Criteria	ASTM D2845-08	ISRM	RMS T224
1.1 Transducer pair used to measure sonic velocity			
1.2 Method for calculating V_p			
1.3 Method for calculating V_s			
1.4 Shear piezoelectric elements recommended for determination of V_s			
1.5 Includes elastic constant equations			
1.6 Includes quantified measure of sample anisotropy			

Both these standards make provision for the measurement of the compression wave sonic velocity – first arrival. And the measurement of the shear wave sonic velocity – second arrival. Both standards suggest but do not require shear transducers to be used to measure the V_s.

Figure 5 presents a comparison – of the oscilloscope view – between the acoustic wave measured by compression and shear transducer pairs on a trial (sandstone) sample prior to testing on the Alpala geotechnical sample set. Figure 5a presents an acoustic waveform as measured by a compression transducer pair. The first arrival measured as 52μs.

The shear wave is the next expected acoustic wave to arrive or be received by the receiver transducer. However, the second arrival is difficult to discern due to the natural heterogeneity and microstructure of the rock. The second arrival (shear wave) was measured using shear transducers and was found to have a transit time of 82μs. This second arrival would be impossible to discern in this scenario from compression transducers only as there is no clear deviation in the acoustic waveform to indicate that the shear wave front has arrived and been received by the transducer.

Figure 5b presents the acoustic waveform of the trial sample as measured by a shear transducer pair. The 'first' arrival is clearly denoted by the voltage change at 82μs. During acoustic wave velocity measurement, the shear transducer predominately generates a shear wave in the rock sample. This shear wave propagates through the rock sample and is received by a shear transducer that is excited primarily by lateral oscillations caused by shear displacement of the shear wave. Due to the mechanics of the shear wave transducer the 'first' arrival, as depicted in Figure 5b, is the shear wave.

Strictly speaking the arrival of the shear wave is always the second arrival. A very low amplitude or 'parasitic' compression wave is known to be produced by shear wave transducers (Yurikov et. al., 2019). In this scenario the 'parasitic' compression wave is of low amplitude and appears to be registered at approximately 56μs transit time. The point at which the voltage deviates from 0V on the oscilloscope. Figure 5 presents an ideal case where the first arrival, as measured by the compression transducers, represents the compression wave sonic velocity and the 'clear' or 'significant' first arrival measured by the shear transducers represents the shear wave sonic velocity.

RESULTS

Acoustic velocity was measured on ninety (90) samples from the broader Alpala deposit geotechnical testing programme. The geotechnical testing was conducted by STS to the relevant Australian Standards (AS) unless otherwise stated.

All 90 samples had compression wave sonic velocity measured using the RMS T224 test method. 56 samples had shear wave sonic velocity measured using shear transducers under the RMS T224 test method framework, modified by using shear transducers. These 56 samples were all subject to destructive UCS testing (AS4133.4.2.1 and AS4133.4.2.2), during which Young's modulus and Poisson's ratio was measured to AS4133.4.3.1 and AS4133.4.3.2.

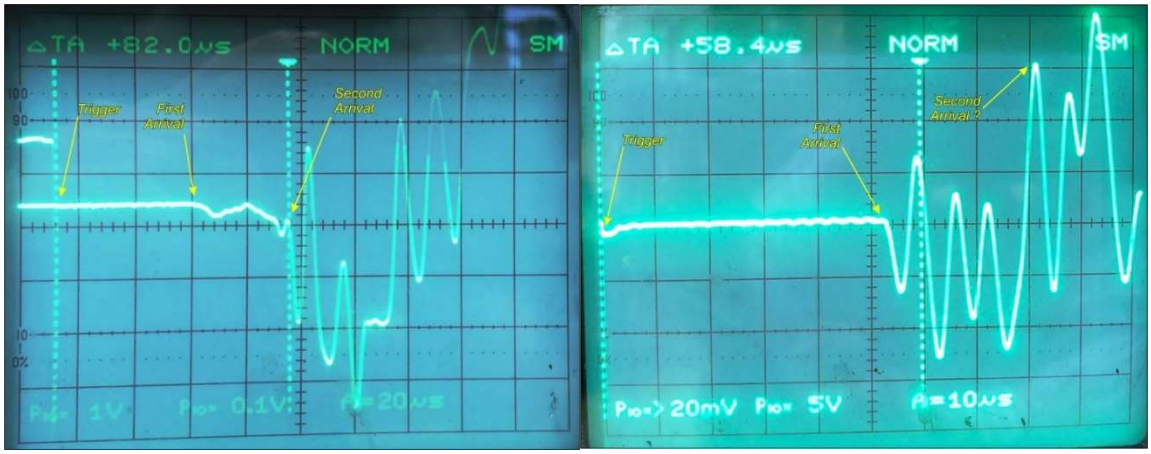

Figure 5: Oscilloscope view of the acoustic wave measured by compression (top) and shear (bottom) transducers

Acoustic wave measurements

Figure 6 presents the acoustic wave velocities – compression and shear wave – from the hard rock dataset consisting of various metamorphic and igneous rocks.

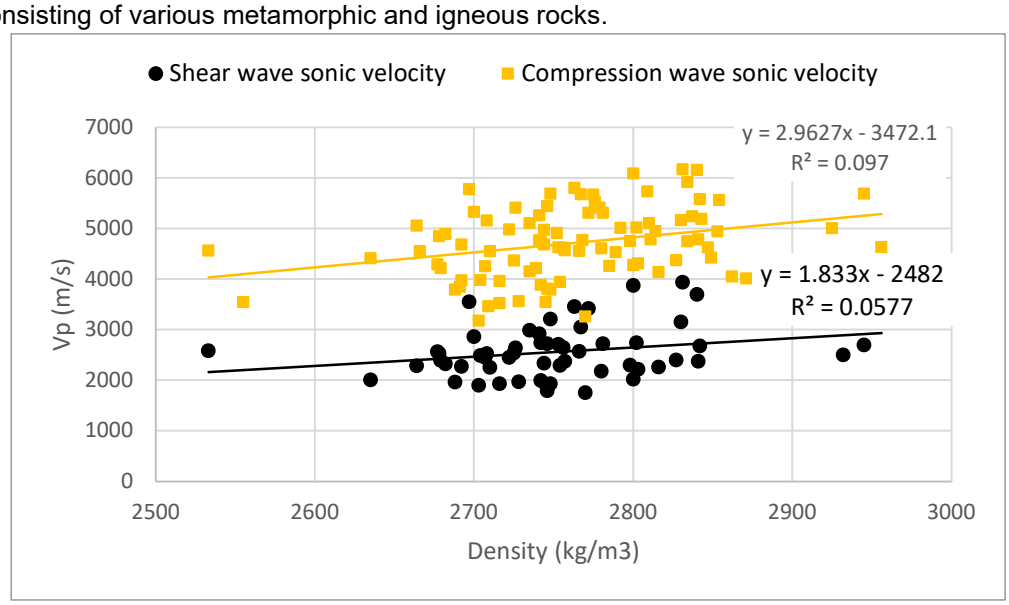

Figure 6: Laboratory P and S wave sonic velocities measured relative to rock density

Figure 6 illustrates the faster acoustic velocity of the compression wave through the rock sample in comparison to the slower shear wave acoustic velocity. The average V_p/V_s ratio was 1.87. This result is consistent with the literature which states the P wave is the first arrival and the S wave is the second arrival of the acoustic waveform, mathematically represented by $(V_p) > (1.41 * V_s)$ (Bassiouni, 1994).

Shear wave sonic velocity by equations of motion

Figure 7 presents the shear wave sonic velocity measured on 56 hard rock samples against measured density as well as the sonic velocity calculated by equations of motion [Equation 3.19 (Bassiouni, 1994)]. Figure 7 shows the trend between V_s and density is strongly correlated for both the calculated and measured values. V_s measured values are slightly greater in magnitude than the V_s values calculated by equations of motion.

Figure 7 highlights the relatively extreme high and low V_s values calculated by equations of motion. The grouping of the V_s measured values are much more consistent and therefore the function is more representative of the dataset. This is driven by the properties which are used to calculate the V_s value in [Equation 3.19 (Bassiouni, 1994)]. These properties include Young's modulus, Poisson's ratio and density.

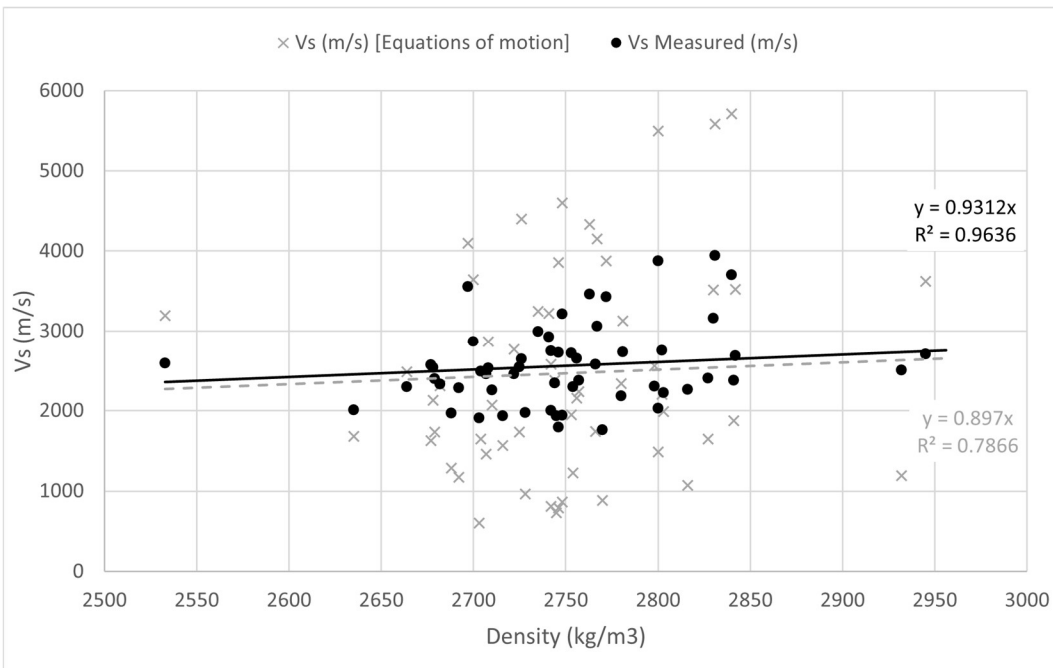

Figure 7: Measured and calculated shear wave sonic velocity relative to density

The theoretical equation assumes the rock material is homogenous and isotropic, which makes it sensitive to any microstructure or inclusions that may be naturally occurring.

Shear wave sonic velocity by Christensen's Equation

Figure 8 presents the shear wave sonic velocity measured on 56 hard rock samples against measured density as well as the sonic velocity calculated by Christensen's equation. Figure 8 shows the trend between V_s and density is strongly correlated for both the calculated and measured values. The function defining V_s measured is 11% greater than the function defining V_s calculated. This indicates that V_s measured values are slightly greater in magnitude than the V_s values calculated by Christensen's equation.

Christensen's equation is an empirical derived equation commonly used in hard rock environments to calculate shear wave sonic velocity (Firth and Elkington, 1999). Natural variation between rock types is expected due to grain size, mineral concentration, grain boundaries and inclusions. Variation is also expected in the microstructure of samples due to the relative stress fields the rocks were subject to and the degree of micro fracturing that may have occurred. These natural variation are expected to manifest as variation between the empirical equation results and measured shear wave sonic velocity results. This outcome is evident in Figure 8.

The two methods for calculating V_s independently validate the accuracy and appropriateness of the measured shear wave sonic velocities presented in this paper. The equations of motion assume a homogenous, isotropic, elastic medium. The shortfalls of the theoretical calculation when applied to rock samples have been discussed and are illustrated by the relatively extreme variations in high and low calculated shear wave sonic velocities. Christensen's equation is commonly used to calculate shear wave sonic velocity in hard rock environments, relying on density and compression wave sonic velocity logs to estimate V_s. Natural geological variations between datasets are expected to manifest as variations between calculated and measured shear wave sonic velocity as discussed.

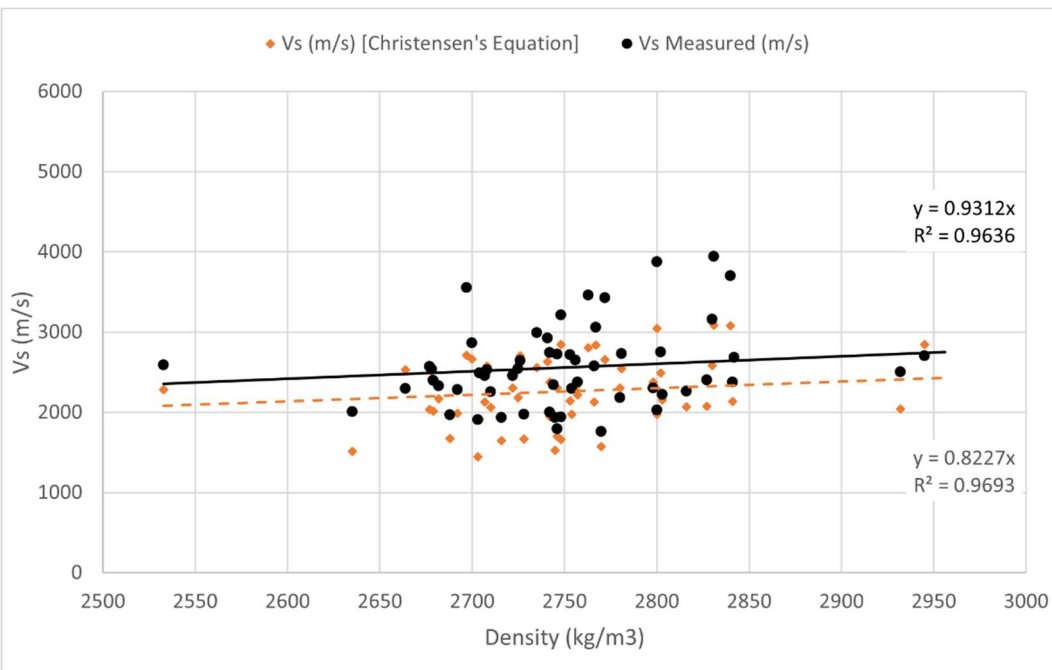

Figure 8: Measured and calculated shear wave sonic velocity relative to density

Static vs Dynamic Moduli

Young's modulus (E)

Figure 9 presents the Young's modulus values measured during UCS testing (static) and sonic velocity measurements (dynamic). Dynamic Young's modulus was found to be on average 28% greater than the static Young's modulus. There is a strong correlation between the static and dynamic Young's modulus measurements.

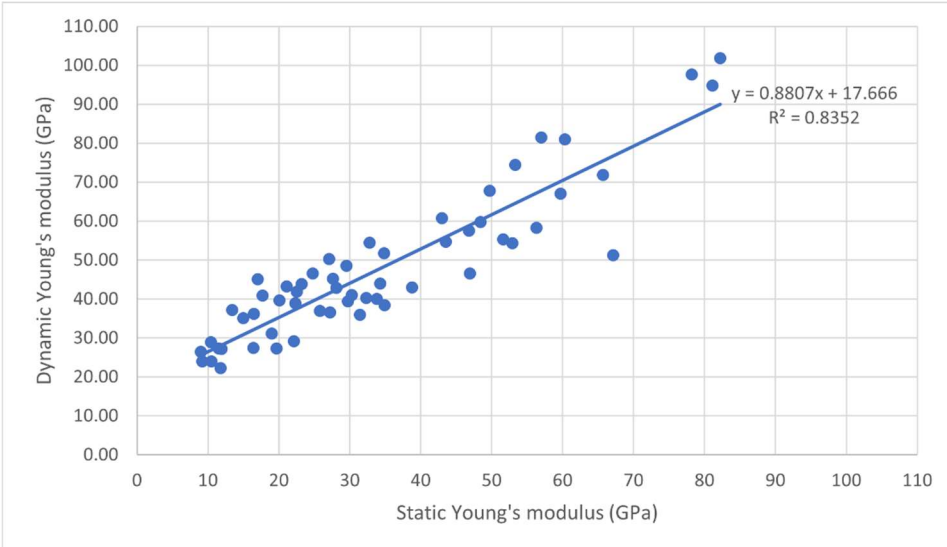

Figure 9: Static versus dynamic Young's modulus for various hard rock lithologies

This suggests that the mathematical relationship y = 0.88x + 17.67 links static and dynamic Young's modulus – for this dataset – and that geotechnical rock properties can be calculated from acoustic velocity measurements. Acoustic velocity measurements of rock would be able to further supplement multiple geotechnical applications.

Figure 10 presents a box plot of dynamic and static Young's modulus values. Dynamic Young's modulus magnitudes are generally greater than static Young's modulus magnitudes for the same sample set.

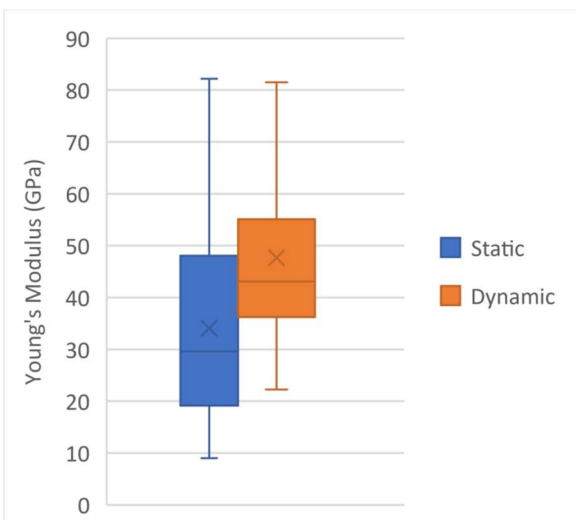

Figure 10: Box plot of dynamic and static Young's modulus (outliers removed).

Dynamic Young's modulus values present a smaller spread of data. This is quantified by end members and a smaller interquartile range in comparison with static Young's modulus values.

Static Young's modulus was measured using two axial strain gauges placed at opposing sides of the middle of the rock sample, consistent with AS4133.4.2.1 and AS4133.4.2.2. This discrete measure of strain change during axial loading is used to represent the axial strain change of the entire sample.

Dynamic Young's modulus is calculated by Equation 3. This equation incorporates compression and shear wave sonic velocity – a measure of elastic properties of the entire rock sample. Equation 3 also includes density, another measure of the properties of the entire rock sample.

Poisson's ratio (ν)

Figure 11 presents the Poisson's ratio values measured during UCS testing (static) and sonic velocity measurements (dynamic). The relationship between dynamic and static Poisson's ratio is not well correlated. The function defining the relationship between dynamic and static Poisson's ratio simultaneously underestimates and overestimates values.

Figure 12 presents a box plot of dynamic and static Poisson's ratio values. Dynamic Poisson's ratio magnitudes are generally greater than static Poisson's ratio magnitudes for the same sample set.

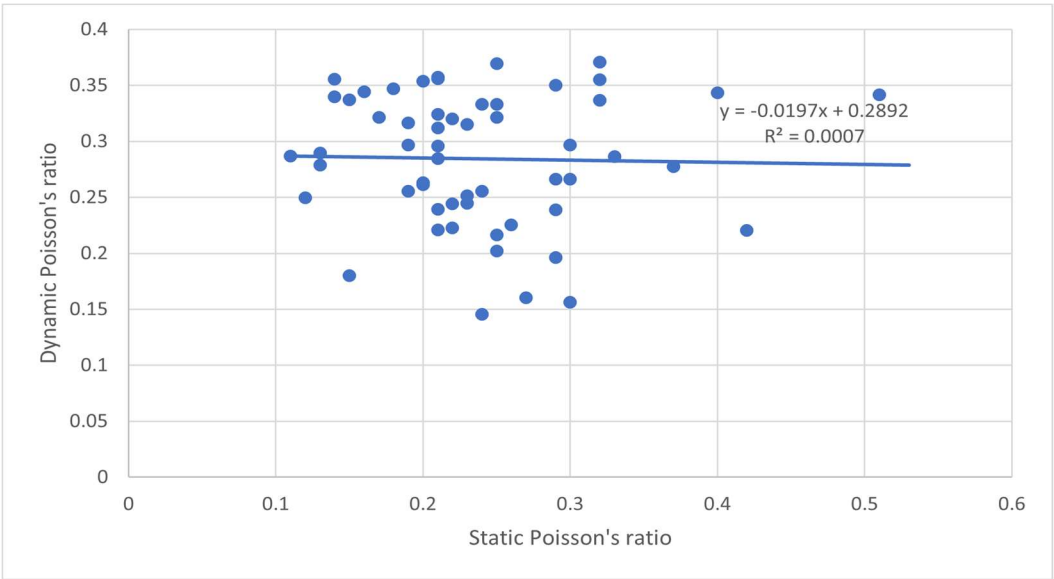

Figure 11: Static versus dynamic Poisson's ratio for various hard rock lithologies

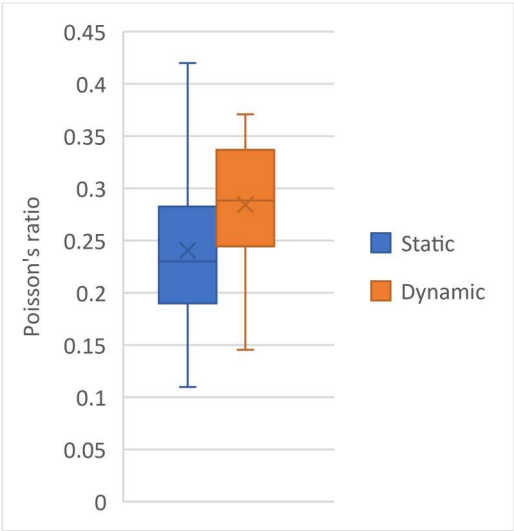

Figure 12: Box plot of dynamic and static Poisson's ratio (outliers removed).

Dynamic Poisson's ratio values present a smaller spread of data. As was the case with Young's modulus values presented in Figure 10. Dynamic Poisson's ratio end members 0.15 and 0.37 are considerably more confined than static Poisson's ratio end members of 0.11 and 0.42.

Static Poisson's ratio was measured using two partial circumferential strain gauges, in addition to the axial strain gauges. These strain gauges were positioned at opposite sides of the middle of the rock sample. Consistent with AS4133.4.2.1 and AS4133.4.2.2.

Shear modulus (G)

Figure 13 presents the shear modulus values measured during UCS testing (static) and sonic velocity measurements (dynamic). Dynamic shear modulus was found to be 30% greater than the static shear modulus. There is a strong correlation between the two methods of shear modulus measurement. For isotropic elastic materials, the shear modulus is directly related to Young's modulus and Poisson's ratio, as given by Equation 3.

The results in Figure 13 suggest that the relationship between dynamic and static shear modulus can be mathematically defined. Suggesting shear modulus can be reliably quantified from acoustic velocity measurements in rock. The greater geotechnical application will be discussed at the end of this paper.

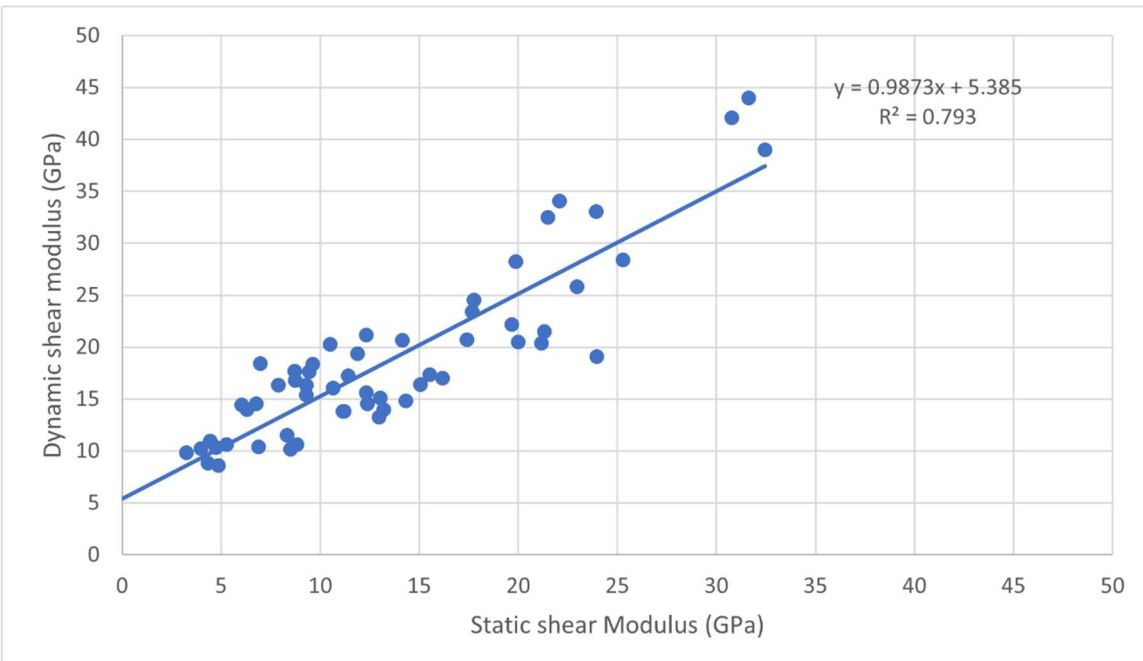

Figure 13: Static versus dynamic shear modulus (G) for various hard rock lithologies

DISCUSSION

Acoustic velocity measurement

No Australian standard (AS) exists for measuring the sonic velocity of rock in a laboratory setting, despite this test being routinely conducted. Two benchmark standards are known to exist for determining the sonic velocity of rock by ultrasonic pulse transmission technique. These standards have been produced by:

- The International Society for Rock Mechanics (ISRM) – Upgraded ISRM Suggested Method for Determining Sound Velocity by Ultrasonic Pulse Transmission Technique
- American Society of Testing Methods (ASTM) – ASTM D2845-08

Australian laboratories are known to use a range of methods for determining the sonic velocity of rock which include:

- RMS T224,
- ASTM D2845-08, and
- In-house test methods.

A benchmark Australian Standard for measuring the sonic velocity of rock is essential. An Australian Standard would result in consistent, accurate and comparable results across laboratories and resources. This would benefit the mining industry by ensuring high quality results that feed into downflow workstreams – dynamic moduli, inferred UCS. This, in turn, will result in higher confidence geotechnical characterisation and aid in reducing geotechnical risk.

This paper puts forward evidence that in order to maximise the success rate of measuring the shear wave sonic velocity, the use of shear transducers should be mandatory. The use of shear transducers would aid in producing a clearly distinguishable second arrival from the low amplitude compression waveform. The objective being to remove interpretation error and produce accurate, repeatable results.

Geotechnical engineering application

The laboratory results presented in this paper have shown that acoustic velocity measurements can be used to calculate dynamic moduli that are mathematically related to static moduli. Specifically, Young's modulus and shear modulus.

The geotechnical applications of acoustic velocity measurements in rock are significant. This non-destructive testing method would allow a greater number of samples to be tested at a lower cost than destructive strength testing methods. Compressional and shear wave velocity data from borehole logging can be used to add to the core-based data set.

There is also the potential of generating three-dimensional (3D) geological models which incorporate rock mass deformation properties using Surfer, Voxler, FLAC3D and other geotechnical modelling packages. This can be achieved by measuring the acoustic velocity of rocks in the field via downhole geophysical surveys. Incorporating the acoustic velocity from a density log and calculating the various dynamic moduli from equations presented in this paper (Bassiouni, 1994). These dynamic moduli logs can be correlated with adjacent boreholes in order to generate a 3D rock mass model.

These measurements and type of analysis are already being conducted but not as frequently as is realistically achievable in the Australian mining industry. The 3D models derived from acoustic velocity measurements would be significant in supplementing current geotechnical characterisation in resources. Geophysical logs allow for measuring deformation moduli continuously along boreholes, adding to the point data derived from UCS and triaxial core testing.

Static and dynamic moduli are expected to vary due to the order of magnitude of stress the rock is subject to during deformation measurement. Further research would be necessary to review the datasets which currently exist, linking static to dynamic moduli – measured in the field. This would guide and facilitate the application of acoustic velocity measurements to obtain deformation properties for use in 3D geotechnical models.

CONCLUSIONS

- The laboratory measurement of shear wave sonic velocity in rock has been successfully completed using shear transducers and the RMS T224 test method framework.
- The shear wave sonic velocity results have been validated by theoretical and empirical equations.
- A relationship between static and dynamic Young's modulus has been expressed mathematically for the geotechnical dataset presented.
- No relationship was observed between static and dynamic Poisson's ratio.
- An Australian Standard for sonic velocity measurement is required for the benefit of the mining industry.

ACKNOWLEDGEMENTS

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